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ECS332 2012/1 Part II.3 Dr.Prapun

7 Angle Modulation: FM and PM

7.1. Recall that a sinusoidal carrier signal

$$A\cos(2\pi f_c t + \phi)$$

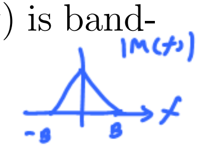
$A(t)$ → Amplitude modulation

$f(t)$ → Frequency Modulation (FM)

$\phi(t)$ → PM

has three basic parameters: amplitude, frequency, and phase. Varying these parameters in proportion to the baseband signal results in amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM), respectively.

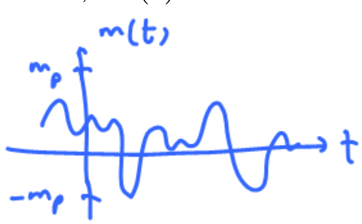
7.2. As usual, we will again assume that the baseband signal $m(t)$ is bandlimited to B; that is, $|M(f)| = 0$ for $|f| > B$.



In this section, we will also assume that

$$|m(t)| \le m_p.$$

In other words, $m(t)$ is bounded between $-m_p$ and m_p.



Definition 7.3. The main characteristic[9] of ***frequency modulation*** is that the carrier frequency $f(t)$ would be varied with time so that

$$f(t) = f_c + km(t), \tag{40}$$

where k is an arbitrary constant.

- The arbitrary constant k is sometimes denoted by k_f to distinguish it from a similar constant in PM.

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7.4. FM: A Magical Technique?

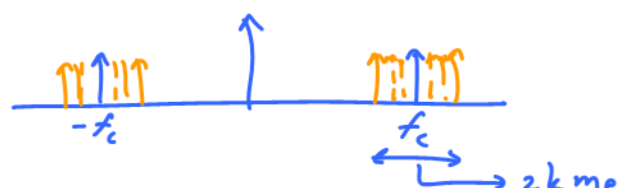
In the 1920s, the idea of frequency modulation (FM) was quite magical. FM was naively proposed very early as a method to conserve the radio spectrum. The naive argument was presented as followed:

- If $m(t)$ is bounded between $-m_p$ and m_p, then the maximum and minimum values of the (instantaneous) carrier frequency would be $f_c + km_p$ and $f_c - km_p$, respectively. (Think of this as a delta function shifting to various location between $f_c + km_p$ and $f_c - km_p$ in the frequency domain.)

$\cos(2\pi f_c t)$

$f(t) = f_c + k \underbrace{m(t)}_{\pm m_p}$

"Not-so-correct" idea



- Hence, the spectral components would remain within this band with a bandwidth $2km_p$ centered at f_c.
- Conclusion: By using an arbitrarily small k, we could make the information bandwidth arbitrarily small (much smaller than the bandwidth of $m(t)$.

In 1922, Carson argued that this is an ill-considered plan. We will illustrate his reasoning later. In fact, experimental results shows that

BW FM $\geq$ BW AM

As a result of his observation, FM temporarily fell out of favor.

[9]Treat this as a practical definition. The more rigorous definition will be provided in 7.11.

7.5. Armstrong (1936) reawakened interest in FM when he realized it had a much different property that was desirable. When the k_f is large, the inverse mapping from the modulated waveform $x_{FM}(t)$ back to the signal $m(t)$ is much less sensitive to additive noise in the received signal than is the case for amplitude modulation. FM then came to be preferred to AM because of its higher fidelity. [1, p 5-6]

7.1 Instantaneous Frequency

To understand more about FM, we will first need to know what it actually means to vary the frequency of a sinusoid.

Definition 7.6. The ***generalized sinusoidal*** signal is a signal of the form

$$x(t) = A\cos\left(\theta(t)\right) \tag{41}$$

where $\theta(t)$ is called the ***generalized angle***.

$A\cos(\omega_c t + \phi)$

- The generalized angle for conventional sinusoid is $\omega_c t + \theta_0$.

7.7. Suppose we want the frequency f_c of a carrier $A\cos(2\pi f_c t)$ to vary with time as in (40). It is tempting to consider the signal

wrong idea

$$A\cos(2\pi f(t)t),$$

where $f(t)$ is the desired frequency at time t.

Example 7.8. See Slides. Consider the generalized sinusoid with $f(t) = t^2$.

Definition 7.9. For generalized sinusoid $A\cos(\theta(t))$, the ***instantaneous frequency***[10] at time t is given by

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t). \tag{42}$$

7.10. Equation (42) implies

$$\theta(t) = 2\pi\int_{-\infty}^{t} f(\tau)d\tau = \theta(t_0) + 2\pi\int_{t_0}^{t} f(\tau)d\tau. \tag{43}$$

[10] Although $f(t)$ is measured in hertz, it should not be equated with spectral frequency. Spectral frequency f is the independent variable of the frequency domain, whereas instantaneous frequency $f(t)$ is a time-dependent property of waveforms with exponential modulation.

Definition 7.11. ***Frequency modulation*** (***FM***):

$$x_{\mathrm{FM}}(t) = A\cos\left(\underbrace{2\pi f_c t + \phi + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau}_{\theta(t)}\right).$$

The instantaneous frequency is given by

$$f(t) = f_c + k_f m(t). \longrightarrow \theta(t)$$

Definition 7.12. ***Phase modulation*** (***PM***):

$$x_{\mathrm{PM}}(t) = A\cos\left(2\pi f_c t + \phi + k_p m(t)\right)$$

The instantaneous frequency is given by

$$f(t) = \frac{1}{2\pi}\theta'(t) = f_c + k_p m'(t)$$

7.13. ***Generalized angle modulation*** (or ***exponential modulation***):

$$x(t) = A\cos\left(\omega_c t + \theta_0 + (m * h)(t)\right)$$

where h is causal.

(a) ***Frequency modulation*** (***FM***): $h(t) = 2\pi k_f 1[1 \geq 0]$

(b) ***Phase modulation*** (***PM***): $h(t) = k_p\delta(t)$.

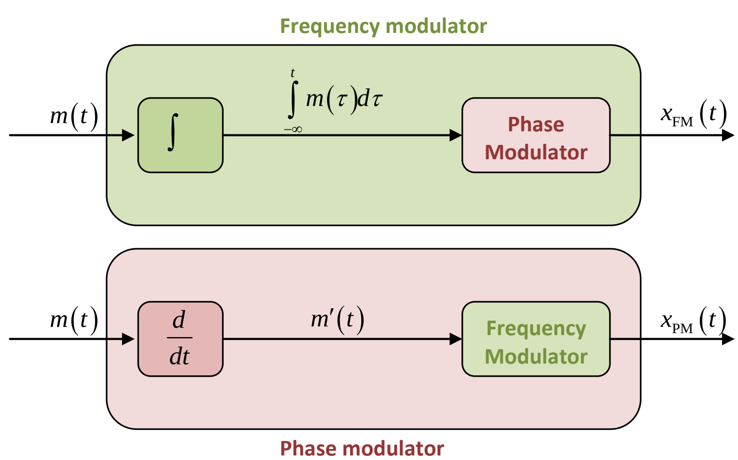


Figure 12: With the help of integrating and differentiating networks, a phase modulator can produce frequency modulation and vice versa [4, Fig 5.2].